

Table 1 The 23 structurally different Solatino patterns in Latin squares of order 5 including the numbers and the colours in which they occur in the puzzle

from: [www.latinsquares5x5.wordpress.com](http://www.latinsquares5x5.wordpress.com) : **How many structurally different latin squares of order 5 do exist?** This table to be used together with *table 2: All 192 structurally different latin squares of order 5...* ([www.latinsquares5x5.files.wordpress.com/2015/07/latintables1.pdf](http://www.latinsquares5x5.files.wordpress.com/2015/07/latintables1.pdf))

Twofold symmetrical patterns with a centre (= central dot)

1						2						3						4					
o						o						o						o					
	o					o							o						o			o	
		o					o					o		o				o		o			
			o					o						o						o			
	1 x black						1 x black						1 x black						1 x black				

Singularly symmetrical patterns with a centre

5						6					
o						o					
			o				o				
	o							o			
		o							o		
o									o		
	2 x black						2 x black				

Asymmetrical pattern with a centre

7					
o					
			o		
		o			
				o	
o					
	4 x black				

Singularly symmetrical patterns without a centre

8	9	10	11
o	o	o	o
o	o	o	o
o	o	o	o
o	o	o	o
o	o	o	o
1 x red 1 x yellow	1 x red 1 x yellow	1 x green 1 x blue	1 x green 1 x blue

12	13	14	15
o	o	o	o
o	o	o	o
o	o	o	o
o	o	o	o
o	o	o	o
1 x red 1 x yellow	1 x green 1 x blue	1 x green 1 x blue	1 x red 1 x yellow

Asymmetrical patterns without a centre

16	17	18	19
o	o	o	o
o	o	o	o
o	o	o	o
o	o	o	o
o	o	o	o
1 x red 1 x yellow 1 x green 1 x blue	1 x red 1 x yellow 1 x green 1 x blue	1 x red 1 x yellow 1 x green 1 x blue	2 x red 2 x yellow

	20		21		22		23	
o			o		o		o	
	o			o		o		o
		o		o		o		o
	o		o		o		o	
	1 x red		1 x red				1 x red	
	1 x yellow		1 x yellow				1 x yellow	
	1 x green		1 x green		2 x green		1 x green	
	1 x blue		1 x blue		2 x blue		1 x blue	

The 23 patterns form in fact a two-dimensional representation of the  $5! = 120$  permutations of the row 12345. Explanation:

Consider pattern 23. This pattern does not contain any symmetry. Pattern 23 in the orientation as shown above can be written as 21453 in the sense that the dot in the upper row stands in position 2, the dot in the second row in position 1, the dot in the third row in position 4, in the fourth row in position 5, in the fifth row in position 3. And there it is: 21453. When you rotate the square  $90^\circ$  clockwise and do the same, you get 45132. Rotating again two times  $90^\circ$  you get 31254 and 43512. Now turn the pattern upside down, and do the same. You get 21534, 45213, 23154 and 35412. In total eight permutations of 12345.

Now consider pattern 15. One of the diagonals forms a symmetry axis. The shown pattern can be written as 24153. Rotation  $90^\circ$  clockwise gives 35142, 31524 and 42513. Turning upside down (= reflection), and then rotation again gives respectively 31524, 42513, 24153 and 35142, the same codes as the former four. Four permutations.

And now pattern 1. Both diagonals form a symmetry axis. It is easy to see that the pattern gives only the codes 12345 and 54321. Two permutations.

We can conclude that the 9 asymmetrical patterns contribute in total an amount of  $9 \times 8 = 72$  permutations to the 120. The 10 singular symmetrical patterns contribute  $10 \times 4 = 40$  permutations, and the 4 two-fold symmetrical pattern contribute  $4 \times 2 = 8$  permutations. In total exactly the 120 permutations of the row 12345. With which has (also) been demonstrated that there cannot be more than 23 Solatino patterns in Latin squares of order 5.